Chapter 7

Imaging With Dipolar Arrays

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In this lecture we will discuss the radio telescopes in which a beamforming network is used to combine signals from the antenna elements and may also provide the required aperture distribution for beam shaping and side lobe control.

7.1 Early History of Dipole Arrays

Radiotelescopes with a variety of antennas of different forms have been built to suit the large range of wavelengths over which radio observations are made[1]. Quasi-optical antennas such as parabolic reflectors are considered more appropriate for milli-meter and centi-meter wavelengths. At the other end of the radio spectrum, multi element arrays of dipole antennas have been preferred for meter and deca-meter wavelengths.

Early observations in radio astronomy were made using one of the two methods, either pencil beam aerials of somewhat lower resolution to investigate the distribution of radio emission over the sky, or interferometers to observe bright sources of small angular size. However, the observations made during the early 1950's, showed that to determine the real nature of the radio brightness distribution it is necessary to construct pencil beam radio telescopes having beam widths of the same order as the separation between the lobes of the interferometers then in use ($\sim 1'$). An important step towards such modern high-resolution radiotelescopes was the realisation that in many cases even unfilled apertures, which contain all the relative positions of a filled aperture, ("skeleton telescopes") can be used to measure the brightness distribution. A cross-type radio telescope, pioneered by Mills was the first to demonstrate the principle of skeleton telescopes.

A cross consists of two long and relatively narrow arrays arranged as a symmetrical cross, usually in the $N-S$ and the $E-W$ directions, intersecting at right angles at their centers (Figure 7.1). Each array has a fan beam response, narrow along its length and wide in a perpendicular direction[2]. The outputs from both the arrays are amplified and multiplied together; only sources of radiation that lie within the cross hatched portion of Figure 7.1(b) produce a coherent signal. Thus an effective pencil beam is produced of

[1] see the illustrations in Chapter 3
[2] See Section 6.2.2

angular size determined solely by the length of the two arrays. A substantial number of telescopes were constructed based on this principle.

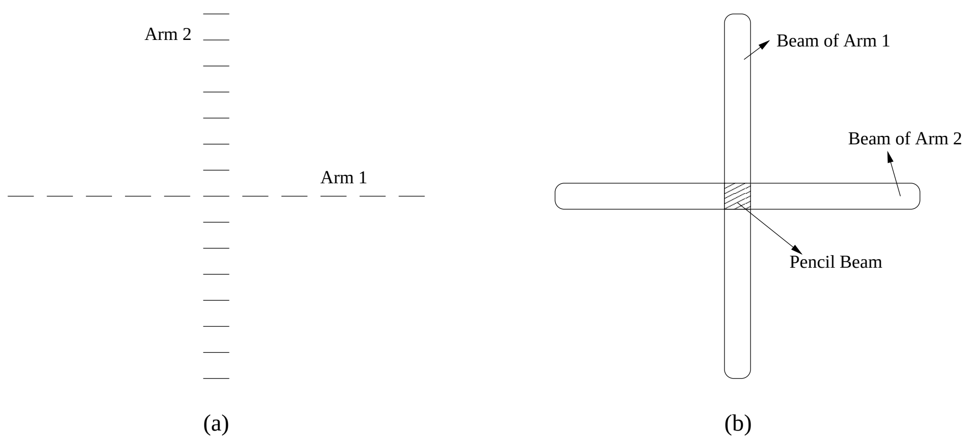

Figure 7.1: A cross type telescope. The arrays in Panel (a) produce the fan beams shown in Panel (b). When the outputs of these two arrays are multiplied together, only signals originating from the cross hatched region common to both beams produce a coherent output. The resolution of such a telescope hence depends only on the lengths of the arms.

The Sydney University telescope was constructed as a cross with aerials of overall dimensions approximately 1 mile long and 40 ft wide (Mills et al 1963). The mile-long reflectors are in the form of cylindrical parabolas, with a surface of wire mesh. Line feeds for two operating frequencies of 408MHz and 111.5MHz were provided at their foci. The $N-S$ arm employs a fixed reflector pointing vertically upwards and the beam is directed in the meridian plane by phasing the dipoles of the feed. The $E-W$ arm is tiltable about its long axis to direct the beam, also in the meridian plane, to intersect the $N-S$ response pattern. No phasing was employed in this aerial. The angular coverage was 55^0 on either side of the zenith. The $E-W$ aperture is divided into two separate halves through which the continuous $N-S$ arm passes. The total collecting area is $400,000$ sq.ft. This instrument had a resolution of approximately $2'.8$ at 408 MHz. This later came to be known as the "Mills Cross" and is one of the earliest cross type radio telescope built. In order to reduce cost, this telescope was built as a meridian transit instrument.

Note that in a cross antenna, one quarter of the antenna provides redundant information, since all element spacings of a filled aperture are still present if half of one array is removed. In fact, it can be shown that the cosine response of a T array is similar to that of a full cross. Thus a survey carried out using a T array has the same resolution as that of a survey carried out using a cross. However it has a collecting area $\sqrt{2}$ times lower than the corresponding cross and hence a lower sensitivity.

7.2 Image Formation

An array can be considered as a sampled aperture. When an array is illuminated by a source, samples of the source's wavefront are recorded at the location of the antenna elements. The outputs from the elements can be subjected to various forms of signal processing, where in phase and amplitude adjustments are made to produce the desired outputs. If the voltages from elemental antennas are simply added (as in the phased arrays discussed in Chapter 6), the energy received from a large portion of the sky will be rejected. When the array is illuminated by a point source this gives the beam of the array which is the Fourier transform of the aperture current distribution. A single beam instrument can use only a part of the total available time to observe each beam width of the sky. One can generate multiple independent beams in the sky by amplifying the signals from element separately and combining them with different phase shifts. Such a multiple-beam or image forming instrument can observe different directions in the sky simultaneously.

A simple linear array, which generates a single beam, can be converted to a multiple beam antenna by attaching phase shifters to the output of each element. Each beam to be formed requires one additional phase shifter per element. Thus an N element array needs N squared phase shifters. Since the formation of a beam is Fourier transforming the aperture distribution, this requirement of N squared phase shifters is very similar to the requirement of N squared multipliers for an N point Fourier transform. Such a network is known as a Blass network (Figure 7.2). Similar to the fast Fourier transform, we also have a Butler beam-forming matrix, which needs only $N \times logN$ elements for beam forming. The Butler matrix uses 90^0 phase-lag hybrid junctions with 45^0 fixed-phase shifters. Blass and Butler networks for a four-element array are shown in the Figure 7.2. If the elemental spacing is $\lambda/2$, the butler matrix produces four beams. Although these beams overlap, they are mutually orthogonal. Surprisingly the Butler matrix was developed before the development of the FFT.

There are a number of drawbacks with multiple-beam formers, viz.

1. It is difficult to reconfigure the beam former. Most multiple beam formers can only produce fixed beams.

2. The separation between the multiple beams cannot be any less than that for orthogonal beams.

3. As the number of beams is increased, one has to keep track of the signal to noise ratio (SNR) of the individual beams.

4. As the array length becomes longer and the total span of the multiple beams increases, the difference between the arrival times of the wave-front from the source to the ends of the array become comparable to the inverse of the bandwidth of the signal used and the loss of SNR due to bandwidth effects becomes large.

7.3 Digital Beam Forming

Digital Beam Forming (DBF) is a marriage between the antenna technology and digital technology. Workers in Sonar and Radar systems first developed the early ideas of digital beam forming. This coupled with the development of aperture synthesis techniques in radio astronomy lead to the development of the modern dipolar arrays.

An antenna can be considered to be a device that converts spatio temporal signals into strictly temporal signals, there by making them available to a wide variety of signal processing techniques. From a conceptual point of view, its sampled outputs represent all of the data arriving at the antenna aperture. No information is destroyed, at least not until the processing begins and any compromises that are made in the processing stages can be noted and estimates made of the divergence of the actual system from the ideal.

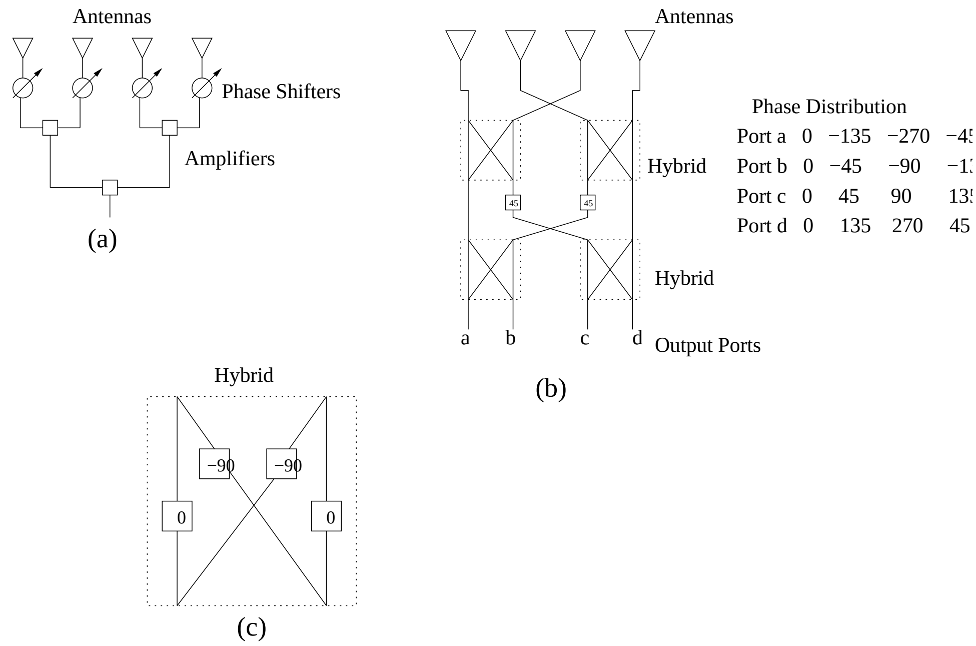

Figure 7.2: A Blass beam forming networks (Panel (a)). Such a network requires N^2 phase shifters to form N beams from N antennas. On the other hand, the Butler beam forming network (Panels (b) and (c)) requires only $N \log(N)$ phase sifters to achieve the same result.

Digital beam forming is based on the conversion of the RF signal at each antenna elements into two streams of binary baseband signals representing cos and sin channels[3]. These two digital baseband signals can be used to recover both the amplitudes and phases of the signals received at each element of the array. The process of digital beamforming implies weighting by a complex weighting function and then adding together to form the desired output. The key to this technology is the accurate translation of the analog signal into the digital regime. Close matching of several receivers is not achieved in hardware, but rather by applying a calibration process. It is expected that more and more of receiver functions will be implemented using software. Eventually one would expect that the receiver would be built using software rather than hardware. We shall get back to this aspect later.

Figure 7.3 depicts a simple structure that can be used for beamforming. The process represented in Figure 7.3(a) is referred to as element-space beamfroming, where the data signals from the array elements are directly multiplied by a set of weights to form the desired beam. Rather than directly weighting the outputs from the array elements, they can be first processed by a multiple-beam beamformer to form a suite of orthogonal beams. The output of each beam can then be weighted and the result combined to produce a desired output. This process is often referred to as the beam-space beamforming (Fig. 7.3(b).

[3]See Section 4.4

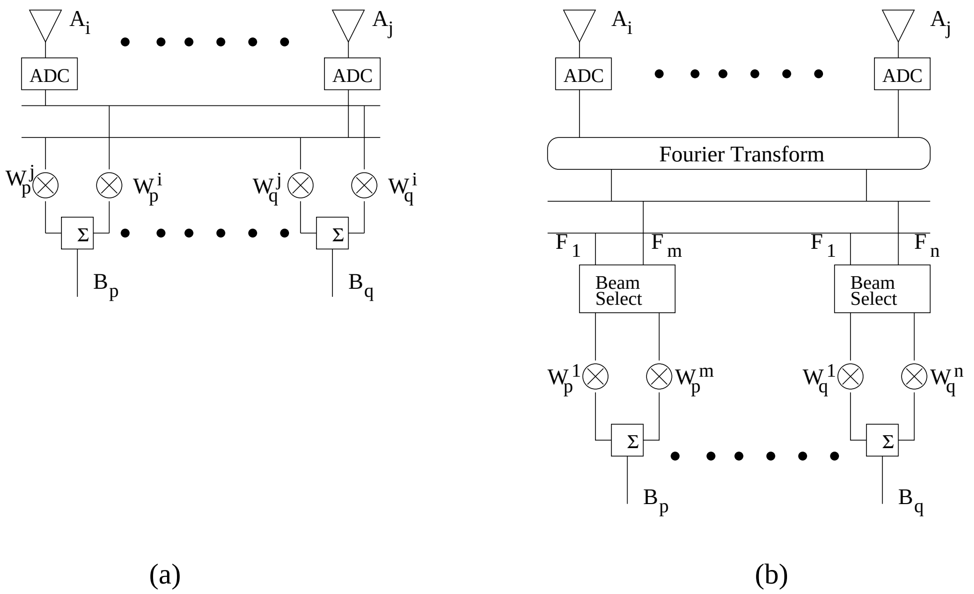

Figure 7.3: Digital beam forming networks. Panel (a) shows an element space beam former while Panel (b) shows a beam space digital beam former.

7.4 Radio Telescopes with Digital Beam Forming Networks

7.4.1 The Clark Lake TEE-PEE-TEE Telescope

This telescope is no more existent. I am using it here as a good example of a telescope which uses a combination of beam forming and synthesis-imaging techniques. This was a fully steerable deca-metric array. This was a T array of 720 conical spiral antennas, 3.0 km by 1.8 km. It had the best sensitivity in the 25 MHz to 75 MHz. Both its operating frequency and beam position were adjustable in less than 1 ms (see Erickson et al. 1982).

The basic element is a long spiral element utilising eight wires wound around a support system that consists of eight parallel filaments. Each element is circularly polarised with a diode switch at its apex that rotates its excitation and thus adjusts its phase. Steering of the array is accomplished by putting a linear phase gradient across groups of 15 elements, called banks. There are 16 banks in the 1800 m $N - S$ arm and 32 banks in the 3000 m $E - W$ arm. The output of each bank is brought separately to the central observatory building.

A separate receiver channel is attached to the output of each of the 48 banks. Each channel employs a superheterodyne receiver[4] to down convert the signal to 10 MHz. The 10 MHz output of each of the receiver channel is sampled at a frequency of 12 MHz digitally delayed and then cross-correlated in a 512 channel two-bit three level complex correlator. An off-line processor removes the fringe rotation[5] introduced by the earth's rotation and integrates the data for periods up to 5 minutes. A Fourier transform then produces a map of the area of the sky under observation. These maps may be averaged to effectively integrate the signal for periods of hours.

It's total collecting area was $250\lambda^2$. The synthesised beam at 30.9 MHz had a width of $13'.0 \times 11'.1$ at the zenith. The confusion limit of the telescope was around 1Jy. It produced

[4]See Section 3.1
[5]See Section 4.4

1024 picture elements in a field of view roughly $6^0 \times 4^0$.

7.4.2 GEETEE: The Gauribidanur T Array

GEETEE is a low frequency radiotelescope operating at 34.5 MHz. It is situated near Gauribidanur, ~ 80 km from Bangalore, India. The antenna system is a T shaped array with 1000 dipoles, 640 in the 1.4 km long $E-W$ array and 360 in the 0.45 km long S array. It's collecting area in the $EW \times S$ correlation mode is 18,000 Sq m and has a resolution of $26' \times 42'\ Sec(\delta - 14^0.1)$. The EW array consists of four rows of dipoles in the NS direction, with 160 dipoles in each row. The S array consists of 90 rows in the NS direction with four dipoles each placed in the EW direction.

A multibeam-forming receiver has been built for $GEETEE$ to obtain long periods of interference free observation over as large a patch of sky as possible in one day. A short observing time for a wide field survey at low frequencies minimises the effects of the ionosphere. For multibeam operation a single row of EW is used in the meridian transit mode. Single row was chosen to maximise the coverage in declination. A single beam in the EW direction was considered sufficient, as the images are confusion limited. 90 outputs of the S array are transmitted to the observatory in 23 open-wire transmission lines using time division multiplexing. In the observatory building, the signals from the EW and S arrays are down-converted to an intermediate frequency of 4 MHz. Then each of the S array output is correlated with the EW array output using one-bit correlators. This gives 90 visibilities sampled at 5 m intervals along the NS direction. The Fourier transform of these visibilities gives 90 multiple beams in the NS direction covering a span of $\pm 47^0$ of Zenith angle along the meridian. A two dimensional image of the sky is obtained by stacking successive scans across the meridian.

7.4.3 MOST: The Molonglo Observatory Synthesis Telescope

A severe disadvantage of the original Mills Cross was that it could make only transit observations. It was recognized that a steerable telescope was necessary to obtain extended observing times and greater sensitivity. To achieve this at a reasonable cost it was decided to abandon the NS arm of the cross and provide a new phased system for the EW arm only. With this a two dimensional aperture is synthesised using earth rotation synthesis. If linear polarisation is used, the position angle of the feeds with respect to the sky will also rotate. Hence, the existing linear feeds were replaced by a circularly polarised feeds.

The usual aperture synthesis procedure accumulates data as points in the spatial frequency (u, v) plane and then interpolates them onto a rectangular grid[6]. The map in the (θ, ϕ) domain is produced by a fast Fourier transform. An important requirement of this method is that the primary beam shape must not vary throughout the observation. This makes it unsuitable for the Molonglo telescope where the primary beam is derived from a rectangular aperture. Because of the mutual coupling problems together with the foreshortening of the effective aperture, the gain of the telescope can vary by over a factor of five as the pointing moves from the meridian to 60^0 from the meridian. This gain variation can be removed from the sampled data, but, the change in beam widths during observations leading to a large variation in the relative gain, between the center of the map and map edges, cannot be corrected for.

The problem of non-circularity and variability of the primary beam may be overcome by the fan beam synthesis or the beam space beam forming. For this the E and the W reflector, each 778 m long and 11.6 m wide (separated by a gap of 15 m) are divided into

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[6]See Chapter 11

44 sections of length 17.7 m. The E and W reflectors are tilted about an EW axis by a shaft extending the whole length. To control the direction of response in an east-west direction a phase gradient is set up between the feed elements by differential rotation. Each module output is heterodyned to 11 MHz. A phase controlled transmission line running the length of each antenna distributes the Local Oscillator. One of these lines is phase switched at 400 Hz.

The detection and synthesis process involves the formation of a set of contiguous fan beams in each antenna. The 44 signals are added together in a resistance array to produce 64 real time fan beams. Signals from corresponding beams from each antenna are multiplied to produce 64 real time interferometer beams. By switching the phase gradient by a small amount every second, these 64 beams are time multiplexed to produce either 128, 256, or 384 beams in each 24-second sample. Each beam has an EW width of 43" and at meridian passage a NS width of $2^0.3$. The hardware beams have a separation of 22" and the time multiplexed beams 11", which is just under half the Nyquist sampling requirement.

If observations of a particular field extend over hour angles of ± 6 h, the fan beam rotates through all position angles and synthesis may be performed. The field is represented by a square array of points corresponding to the projection of the celestial sphere onto a plane normal to the earth's rotation axis. Every 24 seconds, the accumulated signal at each of the 4x63 fan beam response angles are added to the nearest (l, m) array points. This process continues throughout the 12 hours of synthesis. The computation apart from summation includes gain, pointing, and phase corrections; cleaning to improve the map; to locate the sources and to measure their flux densities and position.

7.4.4 Summary

These three radio telescopes illustrate different methods of imaging using dipolar arrays as applied to radioastronomy. GEETEE: One-dimensional image synthesis on the meridian with the entire aperture being present at the same time; CLARK LAKE: A two dimensional image synthesis which gave periods of integration much larger than the meridian transit time. The entire aperture was present during an observation schedule; MOST: Rotational synthesis which is used to synthesise a large two dimensional array, using a linear array. All of them use principles of beam forming. GEETEE and CLARK LAKE use the method of measurement of visibilities in the (u, v) domain, while MOST employs the method of direct fan beam synthesis.

We see that the dipolar arrays are used in the meter wavelength ranges more often than at high frequencies. They have very wide fields of view (GEETEE, almost 100^0) and are very good workhorses for surveying the sky. They are good imaging instruments also since they combine the phased array techniques with the principles of synthesis imaging to make images. Unfortunately most of the arrays are equipped with a limited number of correlators and cannot measure all the possible "$n(n-1)/2$" baselines with "n" aperture elements. Thus they are not well suited for applications of self-calibration. Being skeleton telescopes, they have no redundancy in the imaging mode and redundant baseline calibration is not easily applicable. (See Chapter 5 for a discussion on self-calibration and redundant baseline calibration). This has resulted in surveys with limited dynamic range capability. None of these low frequency arrays are equipped with feeds with orthogonal polarisation. So they are not suitable for polarisation studies.

While combining the beam forming techniques with the synthesis techniques, one has to be very careful about the sampling requirement of the spatial frequencies; otherwise one will end up with grating lobes in the synthesised image, even while using linear arrays with contiguous elements spaced $\lambda/2$ apart. Since the dipolar arrays are employed generally as correlation telescopes and do not have a common collecting area in the arms used for correlation, they suffer from the "zero-spacing problem[7]". Most often today's receivers employ bandpass sampling[8] and if the sampling frequency is not properly chosen one will lose signal to noise. While imaging with arrays it is not un-common, one confronts conflicting requirements between surveying sensitivity and the field of view.

A question may arise in your minds at this stage - with a handful of telescopes using the phased array approach, is there any future for them in radio astronomy? In the remainder of this chapter, I will discuss the possible future of dipolar arrays for radio astronomy.

7.5 Square Kilometer Array (SKA) Concept

In one way or another, all of the various research directions in radioastronomy are limited by our current instrumental sensitivities. Only by ensuring the continued access to order-of-magnitude improvements in our capabilities, can we ensure a continued high rate of discovery! The sensitivity of radio telescopes, in the time between 1940 and 1980, have shown an exponential improvement, over at least 6 orders of magnitude (10^0 mJy to 0.1 mJy for 1 minute integration time). The radio astronomers are toying with the idea of building a telescope with an improvement in sensitivity by a factor of 100 and are hoping that it will lead to fundamental scientific advances (Braun, 1996)

Consideration of the many varied scientific drivers suggests the following basic technical specifications for the instrument:

1. A frequency range of 200 to 2000 MHz.
2. A total collecting area of 1 km^2
3. Distribution over at least 32 elements.

The NFRA in their study of the SKA concept suggest that a broad-band, highly integrated phased array antennas should be adopted for such an array. Some of the advantages are:

1. Phased arrays give "complete" control of beam. The main application considered being the adaptive suppression of RFI environment.
2. Multiple independent beams possible resulting in multiple programs and rapid surveys.

They are planning development work in this direction in several steps: Adaptive array demo, one sq. meter array and a thousand element array and proof of principal arrays. Discussion of all these aspects is beyond the scope of this chapter. Instead we end with the principle of an adaptive array.

7.6 Adaptive Beam Forming

An adaptive beam former is a device that is able to separate signals co-located in the frequency band but separated in the spatial domain. This provides a means for separating the desired signal from interfering signals. An adaptive beam former is able to

[7]The zero spacing problem refers to the difficulty in imaging very large sources, (whose visibilities peak near the origin of the u-v plane) with arrays which provide few to no samples near the u-v plane origin. See Section 11.6 for a more detailed discussion.

[8]See Chapter 1

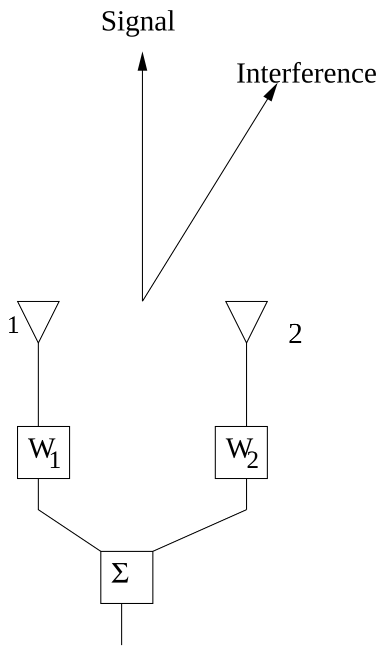

Figure 7.4: A two element adaptive array for interference suppression. The array simultaneously accepts a signal coming from the zenith, while rejecting an interfering signal 30^o from the zenith by a suitable choice of the weights W_i.

automatically optimise the array pattern by adjusting the elemental control weights until a prescribed objective function is satisfied. An algorithm designed for that purpose specifies the means by which the optimisation is achieved. These devices use far more of the information available at the antenna aperture than does a conventional beamformer.

The procedure used for steering and modifying an array's beam pattern in order to enhance the reception of a desired signal, while simultaneously suppressing interfering signals through complex weight selection is illustrated by the following example. Let us consider the array shown in Figure 7.4. The array consists of two antennas with a spacing of $\lambda/2$. Let the signal $S(t)$ arriving from a radio source at zenith is the desired signal. Let $I(t)$ be an interfering signal arriving from a direction $\theta = \pi/6$ radians. The signal from each element is multiplied by a variable complex weight (w_1, w_2) and the weighted signals are then summed to form the array output. The array output due to the desired signal is

$$Y(t) = A\ e^{j2\pi ft}[w_1 + w_2]. \tag{7.6.1}$$

For the $Y(t)$ to be equal to $S(t)$, it is necessary that

$$RP[w_1] + RP[w_2] = 1 \tag{7.6.2}$$

and

$$IP[w_1] + IP[w_2] = 0. \tag{7.6.3}$$

Where RP and IP denote real and imaginary parts of the complex weights. The interfering signal arrives at the element 2 with a phase lead of $\pi/2$ with respect to the element 1. Consequently the array output due to the interfering signal is given by

$$Y_i(t) = [Ne^{j2\pi ft}]w_1 + [Ne^{j2\pi ft+\pi/2}]w_2. \tag{7.6.4}$$

For the array response to the interference to be zero, it is necessary that

$$RP[w_1] + RP[jw_2] = 0 \tag{7.6.5}$$

and

$$IP[w_2] + IP[jw_2] = 0. \tag{7.6.6}$$

The requirement that the array has to respond to only the radio source and not to the interfering signal leads to the solution

$$w_1 = 1/2 - j1/2 \tag{7.6.7}$$

and

$$w_2 = 1/2 + j1/2. \tag{7.6.8}$$

With these weights, the array will accept the desired signal while simultaneously rejecting the interference.

The method used in the above example exploits the fact that there is only one directional interference source and uses the a priori information concerning the frequency and the directions of both of the signals. A more practical processor should not require such a detailed a priori information about the location, number and nature of sources. But this example has demonstrated that a system consisting of an array, which is configured with complex weights, provides numerous possibilities for realising array system objectives. We need to only develop a practical processor for carrying out the complex weight adjustment. In such a processor the choice of the weighting will be based on the statistics of the signal of interest received at the array. Basically the objective is to optimise the beamformer response with respect to a prescribed criterion, so that the output contains minimal contribution from the interfering signal.

There can be no doubt about the worsening observing situation in radio astronomy due to the increased use of frequency space for communications. But a pragmatic view is that it is hopeless to resist the increased use of frequency space by others and we must learn to live with it. The saving grace is that the requirements of mobile cellular, satellite and personal communication services systems are pushing the advancement in technology to provide increasingly faster and less expensive digital hardware. The present trend is to replace the analog functions of a radio receiver with software or digital hardware. The ultimate goal is to directly digitise the RF signal at the output of the receiving antenna and then implement the rest of the radio functions in either digital hardware or software. Trends have evolved toward this goal by incorporating digitisation closer and closer to the antenna at increasingly higher frequencies and wider bandwidths. It is appropriate that the radio astronomer uses this emerging technology to make the future radio telescopes interference free. Adaptive arrays hold the key to this endeavour.

7.7 Further Reading

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