Chapter 9

Correlator – II: Implementation

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The visibility measured by an interferometer is characterized by the amplitude and phase of the fringe at different instants. For simplicity first consider the output of a two element interferometer. In the quasi monochromatic approximation the multiplier output can be written as (see Chapter 4)

$$r_R(\tau_g) = \mathrm{Re}[v_1(\nu,t)v_2^*(\nu,t)] = |\mathcal{V}|\cos(2\pi\nu\tau_g + \Phi_{\mathcal{V}}), \tag{9.0.1}$$

where $v_1(\nu,t)$ and $v_2^*(\nu,t)$ are the voltages at the outputs of the receiver systems of the two antennas, $|\mathcal{V}|$ and $\Phi_{\mathcal{V}}$ are the amplitude and the phase of the visibility and τ_g is the geometric delay. The quantities required for mapping a source are $|\mathcal{V}|$ and $\Phi_{\mathcal{V}}$ for all pairs of antennas of the interferometer. These quantities are measured by first canceling the $2\pi\nu\tau_g$ term in Eq. 9.0.1 by *delay tracking and fringe stopping.* In general, one needs to know the amplitude and phase of the visibility as a function of frequency. This chapter covers the implementation of a spectral correlator to measure the visibility amplitude and phase. Further since the delay tracking (and fringe stopping for some cases) is usually also done by the correlator, these issues are also discussed.

9.1 Delay Tracking and Fringe Stopping

Signals received by antennas are down converted to baseband by mixing with a local oscillator of frequency ν_{LO}. The geometric delay compensation is usually done by introducing delays in the baseband signal. The output of a correlator after introducing a delay τ_i can be written as (see Chapter 4)

$$\begin{aligned} r_R(\tau_g) &= |\mathcal{V}|\cos(2\pi\nu\tau_g - 2\pi\nu_{BB}\tau_i + \Phi_{\mathcal{V}}) && (9.1.2)\\ &= |\mathcal{V}|\cos(2\pi\nu_{LO}\tau_g - 2\pi\nu_{BB}\Delta\tau_i + \Phi_{\mathcal{V}})\ , && (9.1.3)\end{aligned}$$

where ν_{BB} is the baseband frequency and $\Delta\tau_i = \tau_g - \tau_i$ is the residual delay. There are two terms that arise in the equation due to delay compensation:

1. $2\pi\nu_{BB}\Delta\tau_i$, and

2. $2\pi\nu_{LO}\tau_g$.

The first term is due to finite precision of delay compensation and the later is a consequence of the delay being compensated in the baseband (as opposed to the RF, which is where the geometric delay is suffered, see Chapter 4). The phase $2\pi\nu_{BB}\Delta\tau_i$ depends on ν_{BB}. For observations with a bandwidth $\Delta\nu$ this term produces a phase gradient across $\Delta\nu$. The phase gradient is a function of time since the delay error changes with time. The phase $2\pi\nu_{LO}\tau_g$ is independent of ν_{BB}, thus is a constant across the entire band. This phase is also a function of time due to time dependence of τ_g. Thus both these quantities have to be dynamically compensated.

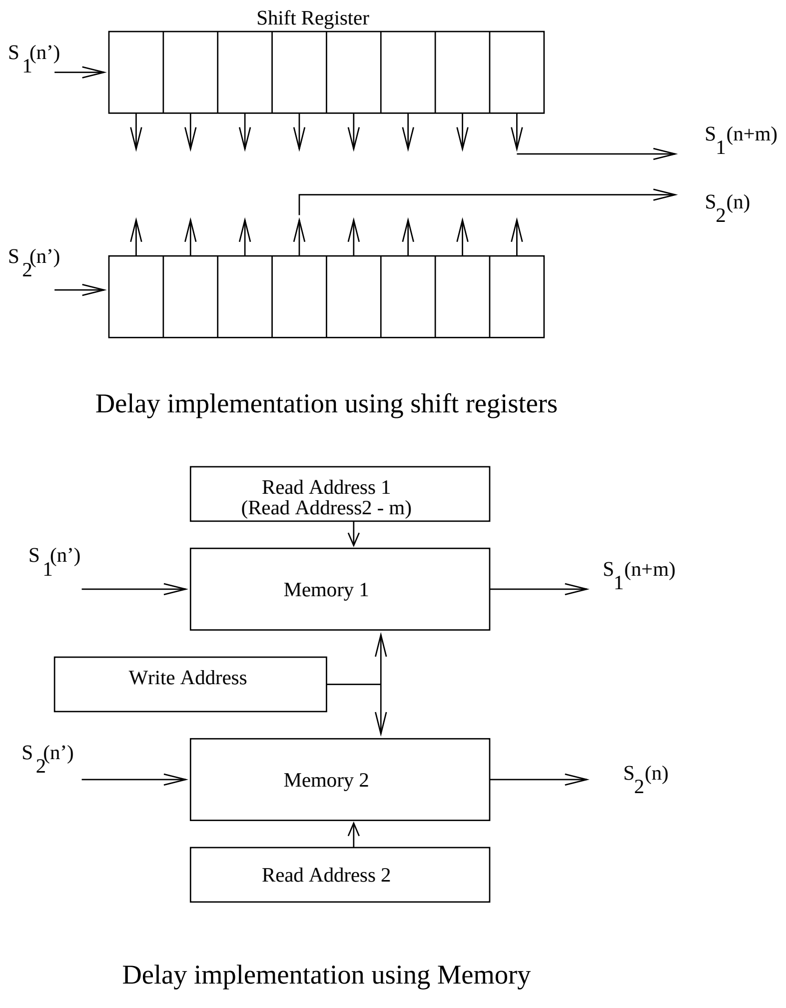

Figure 9.1: Digital implementation of delay tracking in units of the sampling period using shift registers (top) and random access memory (bottom).

Delay compensation in multiples of sampling interval $1/f_s$ can be achieved by shifting the sampled data (see Chapter 8). This is schematically shown in Fig. 9.1. The digitized samples are passed through shift registers. The length of the shift registers are adjusted to introduce the required delay between the signals. Another way of implementing delay is by using random access memory (RAM). In this scheme, the data from the antennas are written into a RAM (Fig. 9.1). The data is then read out from this memory for further proccessing. However, the read pointer and the write pointer are offset, and the offset between the two can be adjusted to introduce exactly the required delay. In the GMRT correlator, the delay compensation is done using such a high speed dual port RAM.

A fractional delay can be introduced by changing the phase of the sampling clock. The phase is changed such that signals from two antennas are sampled with a time difference equal to the fractional delay. A second method is to introduce phase gradients in the spectrum of the signal (see Chapter 8). This phase gradient can be introduced after taking Fourier Transforms of signals from the antennas (see Section 9.2.1).

Compensation of $2\pi\nu_{LO}\tau_g$, (called *fringe stopping*, can be done by changing the phase of the local oscillator signal by an amount ϕ_{LO} so that $2\pi\nu_{LO}\tau_g - \phi_{LO} = 0$. Alternatively, this compensation can be achieved digitally by multiplying the sampled time series by $e^{-j\phi_{LO}}$. (Recall from above that the fringe rate is the same for all frequency channels, so this correction can be done in the time domain). The fringe

$$\phi_{LO}(t) = 2\pi\nu_{LO}\tau_g = 2\pi\nu_{LO}\frac{b\sin(\Omega t)}{c} \tag{9.1.4}$$

is a non-linear function of time (see Chapter 4). Here Ω is the rate at which the source is moving in the sky (i.e. the angular rotation speed of the earth), b is the baseline length and c is the velocity of light. For a short time interval Δt about t_0 the time dependence can be approximated as

$$\phi_{LO}(t) = \phi_{LO}(t_0) + 2\pi\nu_{LO}\frac{b\Omega\cos(\Omega t_0)}{c}\Delta t. \tag{9.1.5}$$

i.e. $\phi_{LO}(t)$ is the phase of an oscillator with frequency

$$\nu_{LO}\frac{b\Omega\cos(\Omega t_0)}{c} \tag{9.1.6}$$

After a time interval Δt the frequency of the oscillator has to be updated. Digital implementation of an oscillator of this type is called a *Number controlled oscillator* (NCO). The frequency of the oscillator is varied by loading a control $number$ to the device. The initial phase of the NCO can also be controlled which is used to introduce $\phi_{LO}(t_0)$. In the GMRT correlator, fringe stopping is done using an NCO.

9.2 Spectral Correlator

The output of a simple multiplier of the two element interferometer after delay compensation can be written as:

$$r_R = |\mathcal{V}|\cos(\Phi_{\mathcal{V}}). \tag{9.2.7}$$

To separate $|\mathcal{V}|$ and $\Phi_{\mathcal{V}}$ a second product is measured after introducing a phase shift of $90\deg$ in the signal path (see Fig 9.2). Introducing a $90\deg$ shift in the path of one of the signals will result in (se Eq. 9.0.1)

$$r_I(\tau_g) = |\mathcal{V}|\cos(2\pi\nu\tau_g + \Phi_{\mathcal{V}} + \pi/2), \tag{9.2.8}$$

and after compensating for $2\pi\nu\tau_g$

$$\begin{aligned} r_I &= |\mathcal{V}|\cos(\Phi_{\mathcal{V}} + \pi/2) \\ &= |\mathcal{V}|\sin(\Phi_{\mathcal{V}}). \end{aligned} \tag{9.2.9}$$

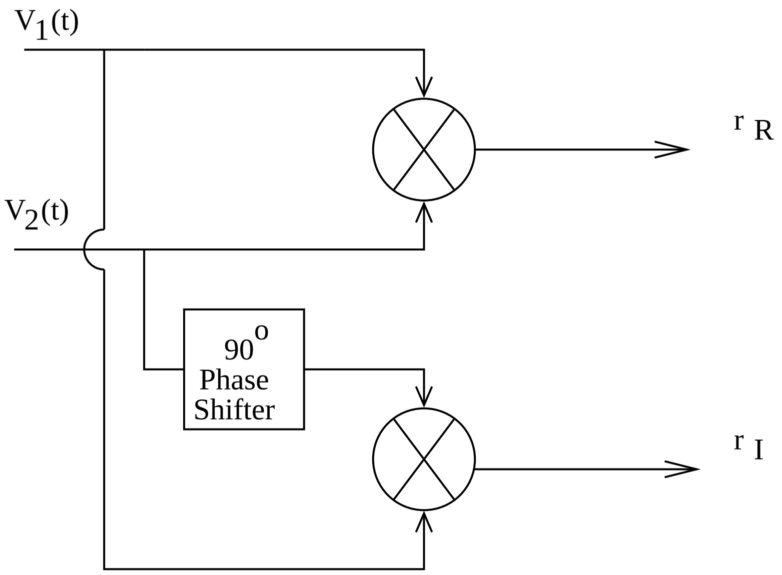

Figure 9.2: Block diagram of a complex multiplier.

From these two measurement we get

$$|\mathcal{V}| = \sqrt{r_R^2 + r_I^2} \tag{9.2.10}$$

$$\Phi_{\mathcal{V}} = \tan^{-1}(\frac{r_I}{r_R}). \tag{9.2.11}$$

Alternatively, for mathematical convenience, the two measurements can be considered as the real and imaginary part of a complex number, i.e.

$$\mathcal{V} = r_R + jr_I \tag{9.2.12}$$

Thus the pair of multipliers together with an integrator (to get the time average) form the basic element of a *complex correlator*.

In the above analysis a narrow band signal (quasi monochromatic) is considered. In an actual interferometer the observations are made over a finite bandwidth $\Delta\nu$ and one requires the complex visibilities to be measured as a function of frequencies within $\Delta\nu$. This can be achieved in one of the two ways described below.

9.2.1 FX Correlator

The band limited signal can be decomposed into spectral components using a filter bank. The spectral visibility is then obtained by separately cross correlating each filter output using a complex correlator (see Fig. 9.3). The digital implementation of this method is called an *FX correlator* (F for Fourier Transform and X for multiplication or correlation). The GMRT correlator is an FX correlator. A schematic of an FX correlator is shown in Fig. 9.4. The analog voltages $V_1(t)$ and $V_2(t)$ are digitized first using ADCs. The geometric delay in steps of the sampling intervals (integral delay) are then compensated for. The

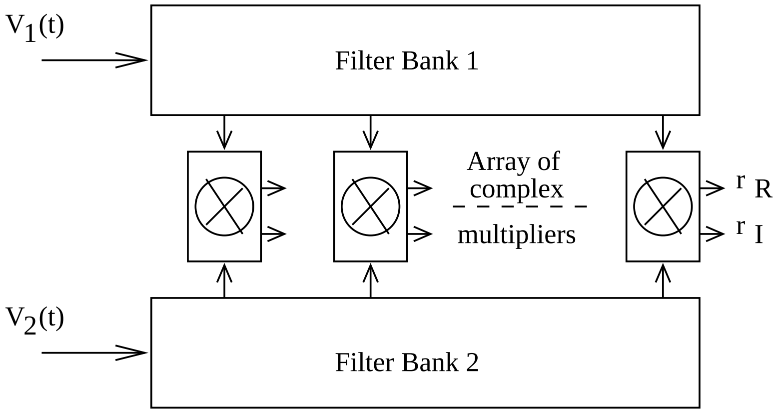

Figure 9.3: A spectral correlator using filter bank and complex multipliers.

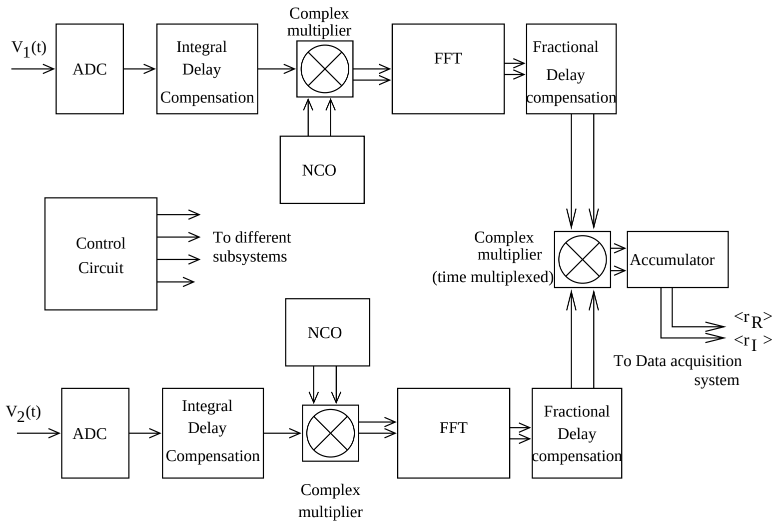

Figure 9.4: Block diagram of an FX correlator.

integral delay compensated samples are multiplied by the output of NCO for fringe stopping. The samples from each antenna are then passed through an FFT block to realize a filter bank. Phase gradients are applied after taking the Fourier Transform for fractional delay compensation. The spectral visibility is then measured by multiplying the spectral components of one antenna with the corresponding spectral components of other antennas. These are then integrated for some time to get an estimate of the cross correlation. Since the Fourier transform is taken before multiplication it is called an FX correlator. For continuum observations with an FX correlator the visibility measured from all spectral components can be averaged after bandpass calibration.

9.2.2 XF Correlator

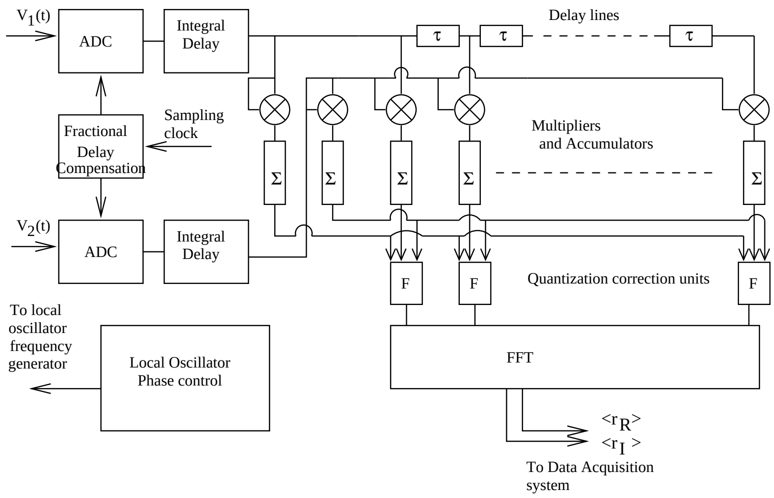

Figure 9.5: Block diagram of a XF correlator.

Eq. 9.0.1 for a broadband signal after delay compensation and integration (time average) can be written as

$$< r_R >= \mathrm{Re}\Big[\int_{-\infty}^{+\infty} < v_1(\nu,t)v_2^*(\nu,t) > \mathrm{d}\nu\Big] \; , \tag{9.2.13}$$

where $v_1(\nu,t)$ and $v_2(\nu,t)$ can be considered as the spectral components of the signals from the antennas. Introducing a delay of τ to one of the signals $v_1(\nu,t)$ modifies the above equation to

$$< r_R(\tau) >= \mathrm{Re}\Big[\int_{-\infty}^{+\infty} < v_1(\nu,t)v_2^*(\nu,t) > e^{-j2\pi\nu\tau}\mathrm{d}\nu\Big] \tag{9.2.14}$$

The above equation is a Fourier Transform equation; the Fourier Transform of the cross spectrum $< v_1(\nu,t)v_2^*(\nu,t) >$ (averaging over t). Thus $< r_R(\tau) >$ is the cross correlation *measured as a function of* τ. Since $v_1(\nu,t)$ and $v_2^*(\nu,t)$ are Hermitian functions, as they are spectra of real signals, their product is also hermitian. Therefore $< r_R(\tau) >$ is a real function and hence it can be measured with a simple correlator (not a complex correlator). Thus the second method of measuring spectral visibility is to measure $< r_R(\tau) >$ for each pair of antennas as a function of τ and later perform an Fourier Transform to get the cross spectrum. The digital implementation of this method is called an *XF correlator*.

A block diagram of an XF correlator is shown in Fig. 9.5. In this diagram, fractional delays are compensated for by changing the phase of the sampling clock. After delay compensation, the cross correlations for different delay are measured using delay lines and multipliers, which are followed by integrator. Since the cross correlation function in general is not an even function of τ, the delay compensation is done such that the correlation function is measured for both positive and negative values of τ in the correlator. The zero lag autocorrelations of the signals are also measured, which is used to normalize the cross correlation. The quantization correction (block marked as F) is then applied to the normalized cross correlations. The cross spectrum is obtained by performing a DFT on the corrected cross correlation function. A peculiarity of this implementation is that the correlations are measured first and the Fourier Transform is taken later to get the spectral information. Hence it is called an XF correlator.

9.3 Further Reading

1. Thompson, R.A., Moran, J.M., Swenson, Jr. G.W., “Interferometry and Synthesis in Radio Astronomy”, Chapter 8, John Wiley & Sons, 1986.

2. Thompson, A.R. & D’Addario, L.R. in “Synthesis Imaging in Radio Astronomy”, R.A. Perley, F.R. Schwab, & A.H. Bridle, eds., ASP Conf. Series, vol. 6.